

# Two-loop corrections to $W$ and $Z$ boson production at high $p_T$

Nikolaos Kidonakis<sup>a</sup> and Richard J. Gonsalves<sup>b</sup>

<sup>a</sup> *Kennesaw State University, Physics #1202, Kennesaw, GA 30144, USA and*

<sup>b</sup> *Department of Physics, University at Buffalo, The State University of New York, Buffalo, NY 14260-1500, USA*

We present new results for the complete two-loop corrections in the soft approximation for  $W$  and  $Z$  boson production at large transverse momentum. Analytical expressions for the NNLO approximate corrections are used to calculate transverse momentum distributions. Results for the  $W$  boson  $p_T$  distribution at Tevatron and LHC energies are presented.

## 1. Introduction

The production of  $W$  and  $Z$  bosons in hadron colliders is important in testing the Standard Model and in the search for new physics, as it is a background to Higgs production and new gauge bosons. Here we present new results for the theoretical calculation of the differential cross section at large transverse momentum. We begin in Section 2 by presenting the leading-order (LO) partonic production channels and presenting some numerical LO results for the  $W$   $p_T$  distribution. In Section 3 we discuss the structure of the NLO corrections and identify an important subset, the soft-gluon corrections. In Section 4, we discuss NNLL resummation of these corrections via two-loop calculations. In Section 5 we present the approximate NNLO  $p_T$  distribution for  $W$  production at the LHC and the Tevatron. We conclude in Section 6 with a summary.

## 2. Partonic channels at LO

The LO partonic processes for  $W$  (or  $Z$ ) production at large  $p_T$  are

$$q(p_a) + g(p_b) \longrightarrow W(Q) + q(p_c)$$

and

$$q(p_a) + \bar{q}(p_b) \longrightarrow W(Q) + g(p_c).$$

We define  $s = (p_a + p_b)^2$ ,  $t = (p_a - Q)^2$ ,  $u = (p_b - Q)^2$ , and  $s_4 = s + t + u - Q^2$ . At the partonic threshold  $s_4 \rightarrow 0$ . As we will see later, the soft-gluon corrections are of the form  $[\ln^l(s_4/p_T^2)/s_4]_+$  while the virtual corrections are  $\delta(s_4)$  terms.

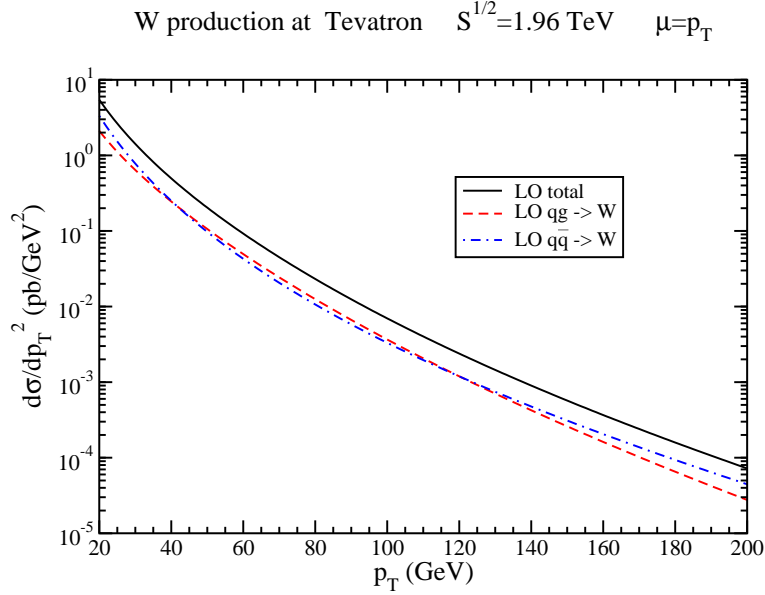
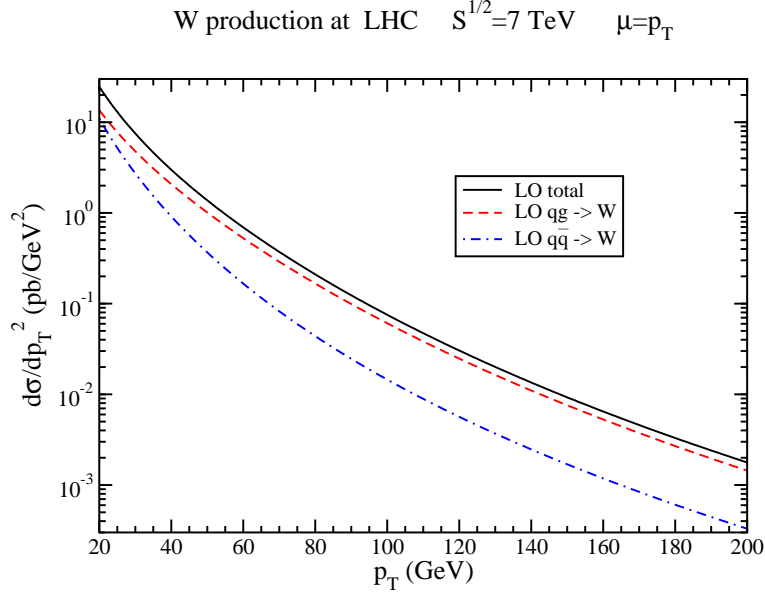
In all numerical results below we use the MSTW2008 NNLO pdfs [1]. We begin with leading-order results for  $W$  production at the Tevatron. In Fig. 1 we show results for the LO  $p_T$  distribution,  $d\sigma/dp_T$ , for  $W$  production at the Tevatron. Here we have set the factorization scale  $\mu_F$  equal to the renormalization scale  $\mu_R$ , and in the numerical results in Fig. 1 we set this common scale, denoted as  $\mu$ , equal to  $p_T$ . We show separate results for the two LO channels as well as for their sum. We note that the  $qg$  and  $q\bar{q}$  channels are equally important. The  $p_T$  distribution falls rapidly as  $p_T$  increases.

Leading-order results for  $W$  production at the LHC at 7 TeV energy are shown in Fig. 2, again with  $\mu = p_T$ . The  $qg$  channel is numerically dominant at this energy for all  $p_T$  values shown in the plot.

The corresponding leading-order results for  $W$  production at the LHC at 14 TeV energy are shown in Fig. 3. Again the  $qg$  channel is numerically dominant.

In Fig. 4 we show the LO scale dependence for  $W$  production at the LHC at 14 TeV. We have chosen  $p_T = 80$  GeV and plot the  $p_T$  distribution as a function of  $\mu/p_T$ . We show three curves. One has the common scale  $\mu = \mu_F = \mu_R$  varied by two orders of magnitude. Another sets  $\mu = \mu_F$  and varies this scale while keeping  $\mu_R$  fixed,  $\mu_R = p_T$ . The third sets  $\mu = \mu_R$  and varies this scale while fixing  $\mu_F = p_T$ . At LO the factorization scale,  $\mu_F$ , and renormalization scale,  $\mu_R$ , dependence largely cancel each other, so that the  $\mu = \mu_F = \mu_R$  line shows relatively mild variation.

In Fig. 5 we show the corresponding LO scale dependence at 7 TeV energy at the LHC. Comparing Figs. 4 and 5 we note a somewhat different scale dependence at 7 and 14 TeV.

Figure 1: LO  $p_T$  distribution for  $W$  production at the Tevatron.Figure 2: LO  $p_T$  distribution for  $W$  production at the LHC at 7 TeV.

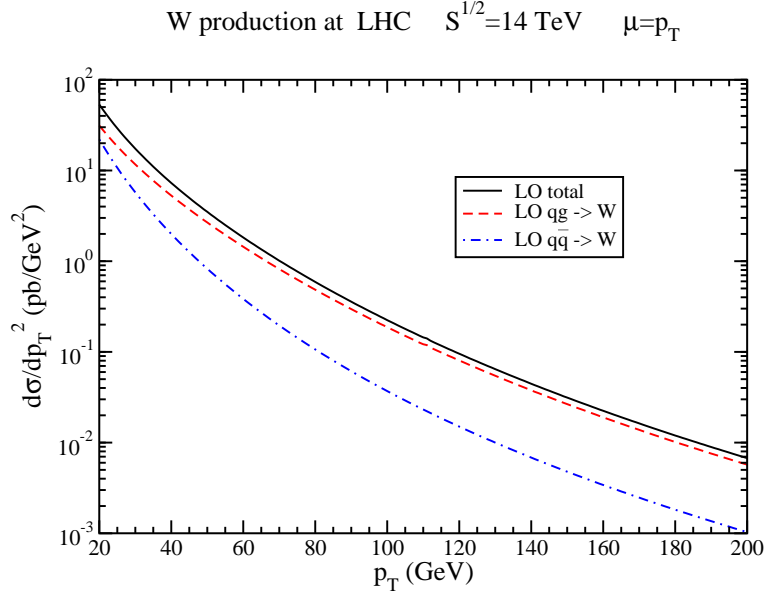
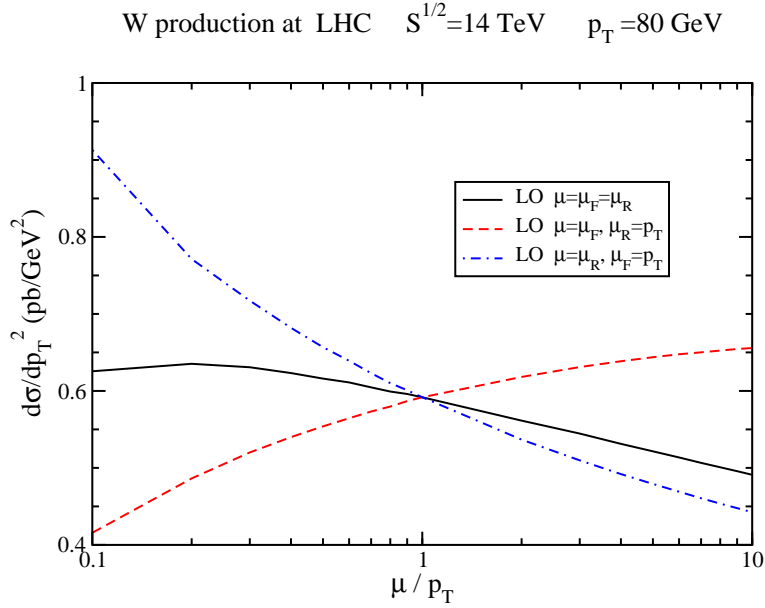
### 3. NLO corrections

The complete NLO corrections were derived in [2, 3]. The NLO cross section can be written as

$$E_Q \frac{d\hat{\sigma}_{f_a f_b \rightarrow W(Q)+X}}{d^3Q} = \delta(s_4) \alpha_s(\mu_R^2) [A(s, t, u) + \alpha_s(\mu_R^2) B(s, t, u, \mu_R)] + \alpha_s^2(\mu_R^2) C(s, t, u, s_4, \mu_F).$$

The coefficient functions  $A$ ,  $B$ , and  $C$  depend on the parton flavors. The coefficient  $A(s, t, u)$  arises from the LO processes.  $B(s, t, u, \mu_R)$  is the sum of virtual corrections and of singular terms  $\sim \delta(s_4)$  in the real radiative corrections.  $C(s, t, u, s_4, \mu_F)$  is from real emission processes away from  $s_4 = 0$ .

The NLO corrections are crucial in reducing theoretical uncertainties and thus making more meaningful comparisons with experimental data for  $W$  production [4–6] and  $Z$  production [7–9] at large transverse momentum.


 Figure 3: LO  $p_T$  distribution for  $W$  production at the LHC at 14 TeV.

 Figure 4: Scale dependence of the LO  $p_T$  distribution for  $W$  production at the LHC at 14 TeV.

At NLO and higher orders terms appear that arise from soft-gluon emission. These soft-gluon corrections are of the form

$$\mathcal{D}_l(s_4) \equiv \left[ \frac{\ln^l(s_4/p_T^2)}{s_4} \right]_+$$

and are numerically dominant for processes near partonic threshold, such as  $W$  or  $Z$  production at large  $p_T$ . For the order  $\alpha_s^n$  corrections  $l \leq 2n - 1$ . At NLO, we have  $\mathcal{D}_1(s_4)$  and  $\mathcal{D}_0(s_4)$  terms, which appear in the analytical expressions for the exact NLO corrections. At NNLO, we have  $\mathcal{D}_3(s_4)$ ,  $\mathcal{D}_2(s_4)$ ,  $\mathcal{D}_1(s_4)$ , and  $\mathcal{D}_0(s_4)$  terms.

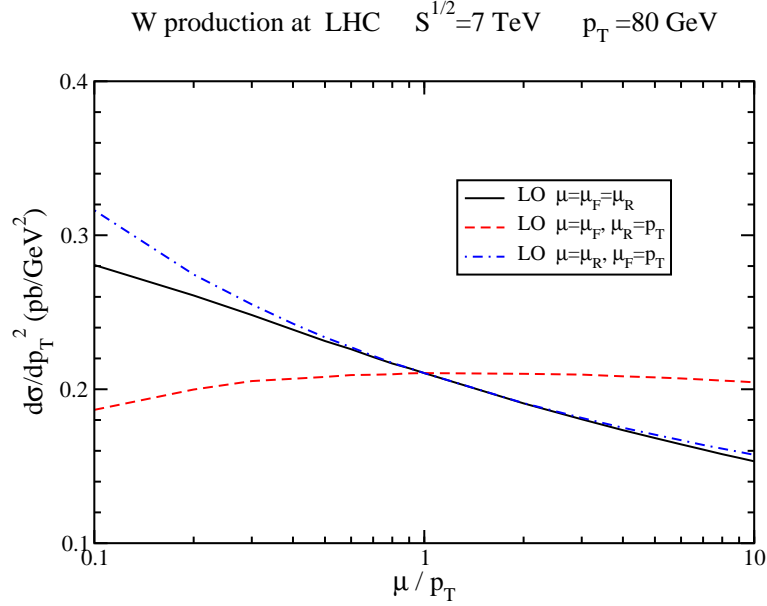


Figure 5: Scale dependence of the LO  $p_T$  distribution for  $W$  production at the LHC at 7 TeV.

#### 4. Two-loop soft-gluon resummation

We can formally resum these soft logarithms for  $W$  and  $Z$  production at large  $p_T$  to all orders in  $\alpha_s$ . This resummation was derived at NLL accuracy in [10] and has been applied to  $W$  production at the Tevatron [11] and the LHC at 14 TeV [12]. In [11] approximate NNLO cross sections were derived from the NLL resummed cross section. After matching with the exact NLO corrections, the NNLO  $\mathcal{D}_3(s_4)$ ,  $\mathcal{D}_2(s_4)$ , and  $\mathcal{D}_1(s_4)$  terms were completely determined. However, only an approximate form of the NNLO  $\mathcal{D}_0(s_4)$  terms was provided. New two-loop results for the soft anomalous dimensions for  $W$  and  $Z$  production [13] now allow us to determine the NNLO  $\mathcal{D}_0(s_4)$  terms fully. We note that recently a different resummation approach, based on Soft Collinear Effective Theory, has also been applied to  $W$  and  $Z$  production at large  $p_T$  [14].

Soft-gluon resummation follows from factorization properties of the cross section, performed in moment space. The resummed cross section is

$$\begin{aligned} \hat{\sigma}^{res}(N) = & \exp \left[ \sum_i E_i(N_i) \right] \exp [E'_j(N')] \exp \left[ \sum_{i=1,2} 2 \int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu} \gamma_{i/i}(\tilde{N}_i, \alpha_s(\mu)) \right] \\ & \times H(\alpha_s) S \left( \alpha_s \left( \frac{\sqrt{s}}{\tilde{N}'} \right) \right) \exp \left[ \int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu} 2 \text{Re} \Gamma_S(\alpha_s(\mu)) \right] \end{aligned}$$

where the first exponential resums the collinear and soft-gluon radiation from the initial-state partons, the second exponential resums corresponding terms from the final state,  $H$  is the hard-scattering function,  $S$  is the soft-gluon function and  $\Gamma_S$  is the soft anomalous dimension.

We expand  $\Gamma_S$  as

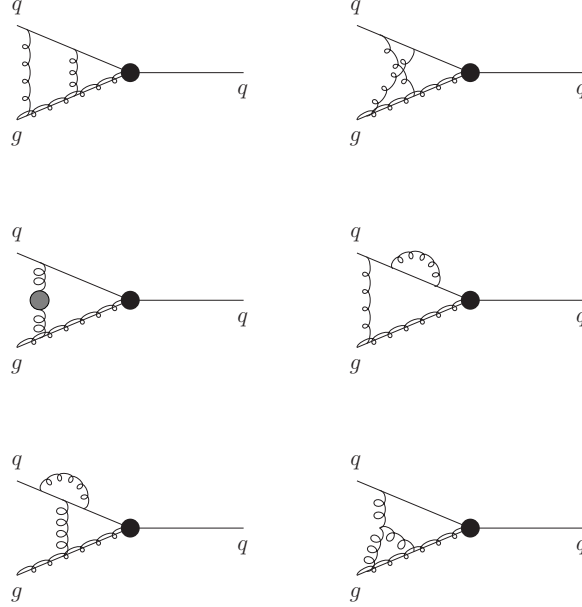
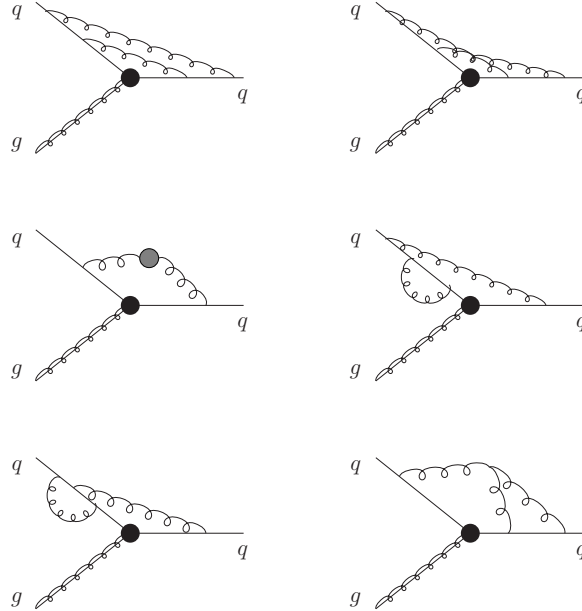
$$\Gamma_S = \frac{\alpha_s}{\pi} \Gamma_S^{(1)} + \frac{\alpha_s^2}{\pi^2} \Gamma_S^{(2)} + \dots$$

The one-loop result,  $\Gamma_S^{(1)}$ , is obtained from the UV poles of one-loop eikonal diagrams, and was first derived in [10].

We determine  $\Gamma_S^{(2)}$  from the UV poles of two-loop dimensionally regularized integrals for the eikonal diagrams shown in Figs. 6, 7, 8.

For  $qg \rightarrow Wq$  (or  $qg \rightarrow Zq$ ) the one-loop soft anomalous dimension is

$$\Gamma_{S,qg \rightarrow Wq}^{(1)} = C_F \ln \left( \frac{-u}{s} \right) + \frac{C_A}{2} \ln \left( \frac{t}{u} \right)$$


 Figure 6: Two-loop eikonal diagrams for  $W$  or  $Z$  production-Set 1.

 Figure 7: Two-loop eikonal diagrams for  $W$  or  $Z$  production-Set 2.

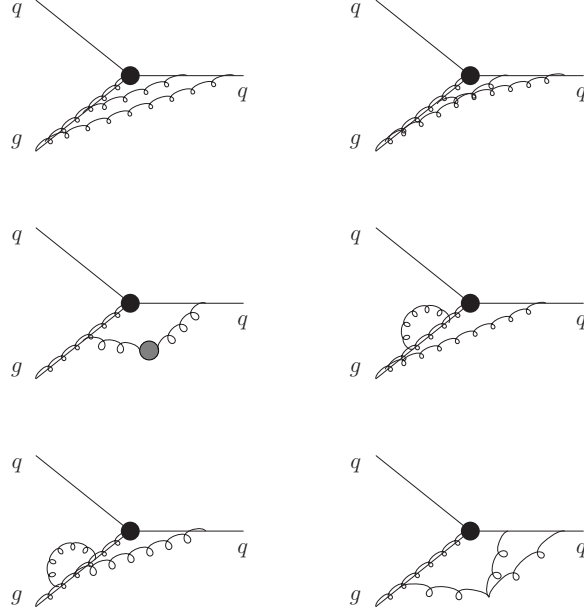
and the two-loop soft anomalous dimension is

$$\Gamma_{S, q\bar{q} \rightarrow Wq}^{(2)} = \frac{K}{2} \Gamma_{S, q\bar{q} \rightarrow Wq}^{(1)},$$

where  $K = C_A(67/18 - \zeta_2) - 5n_f/9$ .

For  $q\bar{q} \rightarrow Wg$  (or  $q\bar{q} \rightarrow Zg$ ) the corresponding results are

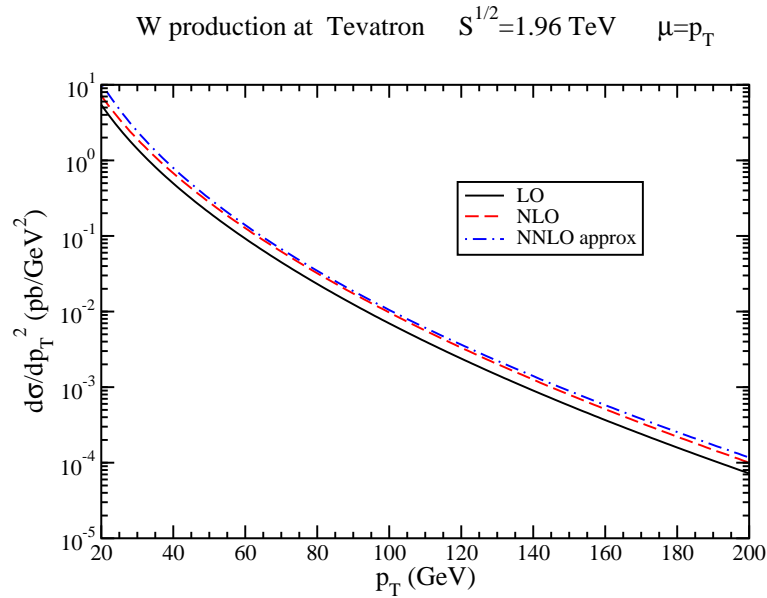
$$\Gamma_{S, q\bar{q} \rightarrow Wg}^{(1)} = \frac{C_A}{2} \ln \left( \frac{tu}{s^2} \right)$$

Figure 8: Two-loop eikonal diagrams for  $W$  or  $Z$  production-Set 3.

and

$$\Gamma_{S, q\bar{q} \rightarrow Wg}^{(2)} = \frac{K}{2} \Gamma_{S, q\bar{q} \rightarrow Wg}^{(1)}.$$

## 5. $W$ production at the Tevatron and the LHC

Figure 9: NNLO approximate  $p_T$  distribution at the Tevatron.

In Fig. 9 we plot LO, NLO, and approximate NNLO results for the  $W$ -boson  $p_T$  distribution at the Tevatron. We have set  $\mu = \mu_F = \mu_R = p_T$ . We note that the NLO corrections are large and the NNLO approximate

corrections are significant.

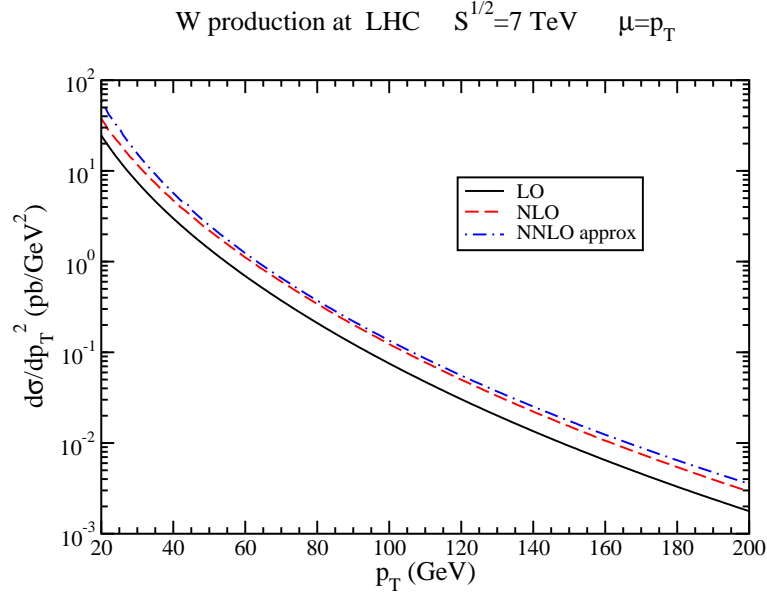


Figure 10: NNLO approximate  $p_T$  distribution at the LHC at 7 TeV.

In Fig. 10 we plot LO, NLO, and approximate NNLO results for the  $W$ -boson  $p_T$  distribution at the LHC at 7 TeV. We note that the NLO corrections are even larger than at the Tevatron and the NNLO approximate corrections are again significant.

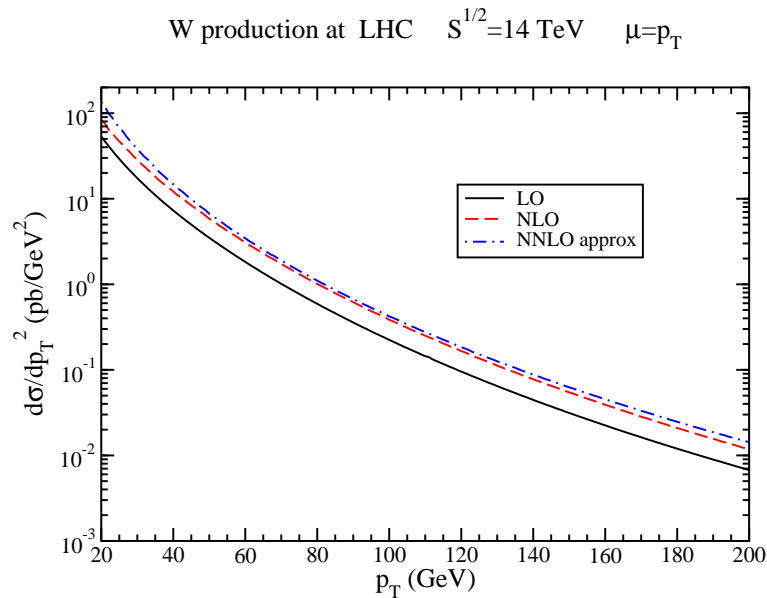


Figure 11: NNLO approximate  $p_T$  distribution at the LHC at 14 TeV.

The corresponding results at 14 TeV energy at the LHC are shown in Fig. 11, with similar conclusions for the size of the higher-order corrections.

## 6. Summary

We have presented new theoretical results for  $W$  and  $Z$  production at large  $p_T$ . We have discussed LO and NLO results and identified the soft-gluon threshold corrections as an important set of corrections. We employed two-loop resummation to calculate NNLO threshold corrections. These corrections are significant and must be included for greater theoretical accuracy. New results for the approximate NNLO  $p_T$  distribution in  $W$  production at the Tevatron and LHC were presented. More work is under way.

## Acknowledgments

This work was supported by the National Science Foundation under Grant No. PHY 0855421.

## References

- 1 A.D. Martin, W.J. Stirling, R.S. Thorne, and G. Watt, Eur. Phys. J. C **63**, 189 (2009) [arXiv:0901.0002 [hep-ph]].
- 2 P.B. Arnold and M.H. Reno, Nucl. Phys. B **319**, 37 (1989); (E) B **330**, 284 (1990).
- 3 R.J. Gonsalves, J. Pawłowski, and C.-F. Wai, Phys. Rev. D **40**, 2245 (1989); Phys. Lett. B **252**, 663 (1990).
- 4 CDF Collaboration, Phys. Rev. Lett. **66**, 2951 (1991).
- 5 D0 Collaboration, Phys. Rev. Lett. **80**, 5498 (1998) [hep-ex/9803003]; Phys. Lett. B **513**, 292 (2001) [hep-ex/0010026].
- 6 ATLAS Collaboration, arXiv:1108.6308 [hep-ex].
- 7 CDF Collaboration, Phys. Rev. Lett. **84**, 845 (2000) [hep-ex/0001021].
- 8 D0 Collaboration, Phys. Rev. Lett. **100**, 102002 (2008) [arXiv:0712.0803 [hep-ex]]; Phys. Lett. B **693**, 522 (2010) [arXiv:1006.0618 [hep-ex]].
- 9 ATLAS Collaboration, arXiv:1107.2381 [hep-ex].
- 10 N. Kidonakis and V. Del Duca, Phys. Lett. B **480**, 87 (2000) [hep-ph/9911460].
- 11 N. Kidonakis and A. Sabio Vera, JHEP **02**, 027 (2004) [hep-ph/0311266].
- 12 R.J. Gonsalves, N. Kidonakis, and A. Sabio Vera, Phys. Rev. Lett. **95**, 222001 (2005) [hep-ph/0507317].
- 13 N. Kidonakis, in *DIS 2011*, arXiv:1105.4267 [hep-ph]; in *DPF-2011*, arXiv:1109.1578 [hep-ph].
- 14 T. Becher, C. Lorentzen, and M.D. Schwartz, arXiv:1106.4310 [hep-ph].